Co-design of traffic network topology and control measures

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Abstract

The two main directions to improve traffic flows in networks involve changing the network topology and introducing new traffic control measures. In this paper, we consider a co-design approach to apply these two methods jointly to improve the interaction between different methods and to get a better overall performance. We aim at finding the optimal network topology and the optimal parameters of traffic control laws at the same time by solving a co-optimization problem. However, such an optimization problem is usually highly non-linear and non-convex, and it possibly involves a mixed-integer form. Therefore, we discuss four different solution frameworks that can be used for solving the co-optimization problem, according to different requirements on the computational complexity and speed. A simulation-based study is implemented on the Singapore freeway network to illustrate the co-design approach and to compare the four different solution frameworks.

Keywords: Network topology design, Dynamic traffic control, Co-design

1. Introduction

In order to improve the performance of a traffic network, the traffic authorities and policy makers usually pose this problem in one of the following two forms: changing the network topology or introducing traffic control measures. Network topology design involves construction work such as building new links or expanding existing links in the network. The advantage of this approach is that it can directly and effectively solve the capacity limitation problem, while the disadvantage is that the implementation may be very expensive and time-consuming, and sometimes the required free space may be not available. On the other hand, traffic control measures aim at a more efficient use of the existing infrastructure, without changing the network topology. However, this approach may be not able to improve traffic flows in some cases, e.g., when the total demand exceeds the capacity of the network. Therefore, we introduce a co-design method that applies the network topology design and the traffic control measures jointly in the traffic network. Intuitively, a better performance of traffic networks is expected by doing co-design of network topology and control measures compared to doing each of them separately. In fact, we will show in this paper that the co-design approach can indeed yield better results in terms of overall costs. Moreover, the co-design approach can be used to assist in the design of

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the network, as it allows to compare different network topologies including variations in types and locations of traffic control measures.

In this paper, we focus on freeway networks, but the co-design approach can also be easily adopted to urban traffic networks. In a freeway network, topology design refers to adding or removing links, or changing the numbers of lanes of links. It seems counterintuitive to remove links or lanes in order to improve the performance of the network, but the fact is that additional road capacity can sometimes induce extra traffic demand, and if not accurately predicted and planned, this extra traffic may lead to the road becoming congested sooner (a well-known example is the Braess paradox (Braess et al., 2005)). Moreover, from an environmental point of view, when freeway networks are built near or through existing communities, the quality of life in the neighborhoods is decreasing due to noise and pollution. In this case, it could be considered beneficial for social reasons to remove or reduce links in the network. In summary, network topology design is a multifaceted problem, where different questions such as environmental impact, budgeting, safety, public inconvenience, etc., have to be considered together. Moreover, some post-design issues such as network maintenance after construction should be also taken into account.

Sometimes, it is not necessary to change the network topology in order to improve the performance of the network, because it is possible that the available infrastructure in the network is not effectively used. Traffic congestion can be caused by the fact that drivers choose routes selfishly or drive in an inappropriate manner. In this case, traffic authorities can introduce traffic control measures to influence driving behavior so that traffic congestion can be eliminated or at least reduced. Papageorgiou et al. (1998) illustrate with a simple example that in a congested area, the total time spent (TTS) in the controlled case is 14% less than in the uncontrolled case, if the traffic outflow is improved by 5% thanks to appropriate traffic control measures. However, this consequence also implies that any disturbance that reduces the traffic outflow with a few percents may significantly increase the TTS, and hence decrease the performance of the network.

While the network topology is not changed once determined for the given design period, the traffic control measures do need to be adapted to the time-varying traffic situations. Due to this different time scale, one usually chooses a standard "optimal" setting of the control strategy, which is static, for the topology design. However, this method is not accurate enough to capture the dynamic nature of the traffic flows in the network. Therefore, we introduce a so-called *parameterized traffic control* approach (Zegeye et al., 2012), where the parameters of the control laws are optimized according to a pre-defined objective function. The reason for using the parameterized control is that for some other control approaches such as optimal control or model predictive control (see Section 2.2.2), the traffic control inputs usually consist of dynamic signals that vary on a minute to minute basis according to the time-varying traffic situations; however, in the parameterized control approach, the parameters of the control laws are considered fixed over the design period, and the control laws generate dynamic traffic signals based on the state of the traffic network. Moreover, we can even consider a more comprehensive quasi-dynamic setting (see Section 4.2), where the design period is divided into different sub-periods, and where each sub-period has a separate group of control law parameters. By using the predicted long-term future evolution of traffic demand, both the topology and the control law parameters can be optimized jointly. It is however important to note that although we use the co-design approach to determine the parameters of control laws for the traffic control measures, this does not mean that these parameters should be fixed for the entire design period. Instead, we can still use online control measures, and regularly retune or optimize the parameters of the control law based on the real traffic situation, via e.g. standard traffic control strategies, optimal control, or model

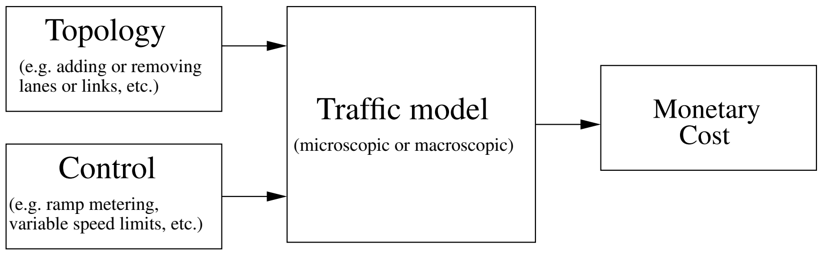


Figure 1: Representative scheme of the co-design problem

predictive control.

The main aim of solving the co-design problem is to find the optimal topology design decisions and the optimal parameters of the control laws. In order to obtain those optimal solutions, both topology design decisions and traffic control measures are applied to a traffic model, and a cost is calculated based on the resulting traffic states (typically traffic density, flow, and speed), and used to evaluate the performance of the traffic system under the impact of topology changes and traffic control measures (see Figure 1). From the traffic management point of view, the objectives of the co-design problem can vary from avoiding traffic congestion to increasing network safety and reliability, and to decreasing fuel consumption and pollution, etc. In this paper, we consider a total monetary cost that includes the budget of construction and maintenance for the network, and a valuation of travel time and travel distance. Note that the construction cost is only spent once, but the maintenance cost is spent every year. Moreover, the price level of the maintenance is not constant but rising every year because of inflation effects. The value of travel time is often used for appraisal of road and public transport projects. It should be included in the monetary cost because it is closely related to the economical factors, e.g., drivers' wages, and interest or depreciation of the freight, etc. A full discussion of value of travel time is out of the scope of this paper, but we refer the interested reader to (DeSerpa, 1971; Wardman, 1998) for more information on this topic. Moreover, travel distance is taken into account in this paper as well because it is also related to the economic factors such as fuel consumption and wear of vehicles.

The main contributions of this paper are:

1. We define a unified problem formulation for co-design of network topology and traffic control measures. We formulate the co-design problem in a model-based optimization framework, where the network topology design and traffic control measures are jointly applied to a traffic model, and a monetary cost is used to evaluate the performance of the traffic network;
2. We discuss four different solution frameworks for solving the proposed co-design problem, namely separate optimization, iterative optimization, bi-level optimization, and joint optimization, according to different requirements regarding performance and computational speed.

The rest of this paper is structured as follows. Section 2 briefly summarizes the state-of-the-art on network topology design and traffic control measures. Section 3 presents the problem statement. We formulate the co-design problem in a mathematical way in Section 4, and propose four different solution frameworks in Section 5. A simulation-based case study of the Singapore freeway network is used to illustrate the proposed approach for solving the co-design problem in Section 6. Finally, we conclude our work and propose some topics for future work in Section 7.

2. State-of-the-art on topology design and traffic control

2.1. Network topology design

According to different forms of design decision variables, the network topology design problem can be posed in a discrete form that deals with changing the number of links or lanes, or a continuous form that deals changing the capacity of links in the network (Yang and Bell, 1998).

2.1.1. Discrete network topology design

The discrete network topology design problem concerns the modification of a traffic network by changing the numbers of links or lanes, and the design decision variables usually have binary or integer values. The objective of the discrete network topology design is to make an optimal investment decision in order to minimize both the total travel cost and total construction cost in the network. Magnanti and Wong (1984) presented a unified view of modeling the discrete network topology design problem, and proposed a unifying framework for describing a number of solution algorithms. Poorzahedy and Turnquist (1982) developed a bi-level programming method, where the lower level aims at finding a user equilibrium flow pattern in the traffic network, and the upper level then determines the design decision variables based on the equilibrium flow resulting from the lower level. Chen and Alfa (1991) studied the lower-level problem by using a logit-based stochastic incremental traffic assignment approach. Gao et al. (2005) proposed a new solution algorithm for the bi-level problem by using the support function concept. One of the challenges for the discrete network topology design problem is that one usually has to solve a nonlinear bi-level mixed-integer optimization problem, which could be extremely computationally complex.

2.1.2. Continuous network topology design

The continuous network topology design problem deals with improvement of the link capacity, and the design decision variables have continuous values. From an implementation point view, there is not much difference between the discrete network topology design and the continuous network topology design. They have the same objective, and can be formulated in the same solution framework, e.g., bi-level programming. However, because the design decision variables are continuous in the continuous network topology design problem, the gradient of the objective function can be obtained, and gradient-based methods can be used to solve the proposed problem (Chiou, 2005). Moreover, sensitivity analysis methods (Tobin and Friesz, 1988; Friesz et al., 1990; Yang, 1997) for the equilibrium network flows can hence also be used.

2.2. Traffic control

Traffic control is a broad topic, so we do not intend to cover all the aspects in this paper. A comprehensive review on this topic can be found in (Papageorgiou et al., 2003).

2.2.1. *Control measures*

We give an overview of control measures that are most frequently used in traffic networks.

- **Ramp metering** determines the flow rate at which vehicles enter the freeway, and is implemented via traffic signals placed on the on-ramp or off-ramp. The vehicles should stop when the light turns red, and they can pass when the light turns green. The purpose of ramp metering is to spread the vehicles that enter the network, and to influence the traffic densities on the mainstream roads in order to prevent a traffic jam or breakdown. Fixed-time ramp metering was adopted at first, but currently dynamic ramp metering is used more often (Papageorgiou and Kotsialos, 2002). ALINEA (Papageorgiou et al., 1991) is one of the best known examples of a dynamic ramp metering strategy.

- **Variable speed limits** can be used to restrict the traffic speed on the freeway. The implementation involves speed limit signs placed over or besides the roads to display the maximum allowed speed for the given stretches of roads. The main purpose is to increase safety by lowering the speed limits upstream of congested areas (Smulders, 1990; Papageorgiou et al., 2008). However, it is also used to improve the traffic flows (Williams, 1996).

- **Route guidance** uses dynamic route information panels or on-board devices to assist drivers in choosing their routes to the destinations. The original purpose is to inform the drivers about the current state of the traffic, e.g. travel time or queue lengths on different routes, to allow the drivers to take corresponding route choice decisions (Mahmassani et al., 2003; Bogers et al., 2005). This method can be also used to persuade drivers to change their route choice in order to obtain a traffic assignment that gives a more optimal traffic performance from the system point of view (Hu and Mahmassani, 1997).

- Other measures include peak lanes that are only open during peak hours, bi-directional lanes that change their direction based on the direction of the highest traffic demand, etc. (Middelham, 2003).

2.2.2. *Control frameworks*

We compare three different traffic control frameworks.

- **Optimal control** aims at obtaining a sequence of optimal control signals based on the system optimum conditions. For a given time horizon N_h, a sequence of control signals $u(0), u(1), \ldots, u(N_\mathrm{h} - 1)$ is determined through minimizing a control objective function subject to constraints. Kotsialos et al. (2002b) used the nonlinear optimal control approach to generate the splitting rates of traffic flows and the ramp metering rates in freeway traffic network. Based on this method, Kotsialos and Papageorgiou (2004) developed a software tool called Advanced Motorway Optimal Control to apply optimal control methods to the traffic in arbitrary-topology freeway traffic networks.

- **Model predictive control (MPC)** can be considered to be optimal control applied in a rolling horizon. The difference is that optimal control has an open-loop structure, while MPC adopts a closed-loop control approach. More specifically, at each control step k_c, MPC determines a sequence of control signals $u(k_\mathrm{c}|k_\mathrm{c}), u(k_\mathrm{c}+1|k_\mathrm{c}), \ldots, u(k_\mathrm{c}+N_\mathrm{p}-1|k_\mathrm{c})$ for the prediction period $[k_\mathrm{c}T_\mathrm{c}, (k_\mathrm{c}+N_\mathrm{p})T_\mathrm{c}]$, with $u(\cdot|k_\mathrm{c})$ the control signal at the corresponding control step based on the information of the control step k_c, T_c the control time length, and

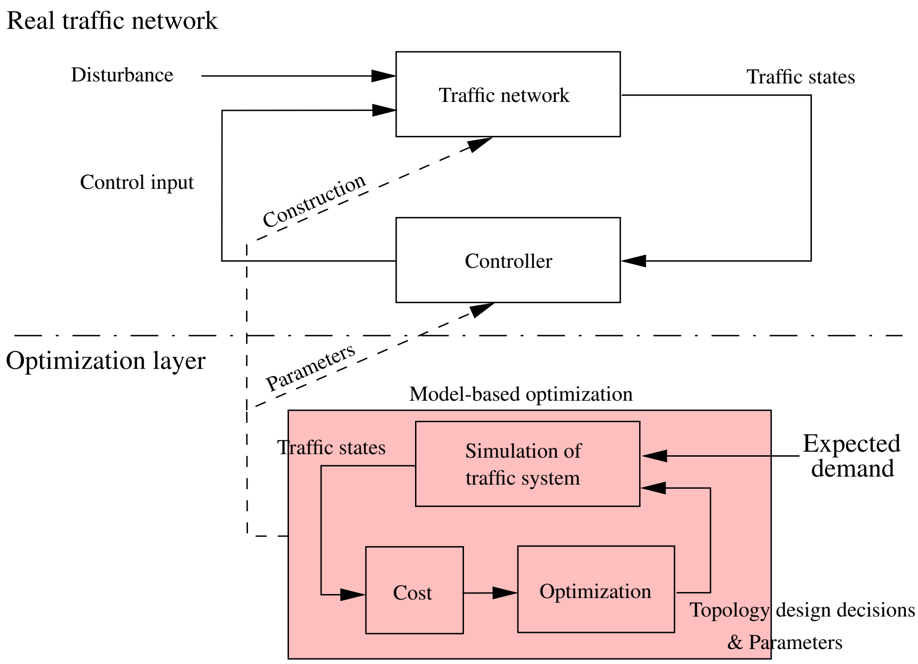


Figure 2: The overall co-design scheme

N_p the prediction horizon. However, only the first control signal $u(k_c|k_c)$ is applied to the system, and then the horizon is shifted to the next prediction period. MPC has been successfully applied to on-line traffic control measures in freeway networks (see Hegyi et al. (2005a); Bellemans et al. (2006)).

- **Parameterized feedback control** aims at finding the optimal parameters of the controller, and the control signals are determined based on the current state of the system via a feedback control law at each control step. ALINEA (Papageorgiou et al., 1991) determines the ramp metering rates via the feedback control law with fixed control parameters. Zegeye et al. (2012) introduce a dynamic framework for parameterized feedback control, containing two layers: the optimization layer and the control layer. At every control step, the optimization layer optimizes and updates the parameters of the control laws, and the control layer then determines the control signals via the control laws. "

3. Problem statement

Figure 2 shows the overall scheme of the co-design approach for determining the optimal network topology and the optimal traffic control strategies, involving two main parts — the real traffic system and the optimization layer.

The real traffic system contains the traffic network and the low-level controllers, such as traffic signals, ramp metering installations, variable speed limits, dynamic route information panels, and so on. The control measures are governed by parameterized control laws. The topology design decisions involve construction work in the network, e.g., building or removing lanes or links. Moreover, installing traffic controllers can also be considered as a topology design issue, e.g., whether to install the controllers, which type of the controllers to choose, and where to locate the controllers.

The optimization layer is used to determine the optimal network topology design decisions and the optimal parameters of the control laws. To do so, the real traffic system is simulated by traffic models in the optimization layer. The traffic simulation is based on:

- the expected daily demand patterns, as well as their long-term forecast,
- the planned topology design decisions and parameters of the control laws.

The performance is evaluated according to the simulated traffic states, and an optimization method is used to determine the optimal solutions. Usually, a traffic network is designed to be used for a long term. However, it will result in an extremely high computational burden to run the traffic simulation and optimization over the entire design period. Therefore, we choose a short range, e.g., one day or one month as a representative *basic design period*. The traffic simulation and optimization only run in the basic design period, but the resulting solutions are applied to the real traffic system for the entire design period, or for different time slots of the entire design period in the quasi-dynamic parameterized control approach (see Section 4.2).

4. Mathematical formulation

4.1. Network topology design

A vector $\boldsymbol{\delta}$ is defined as the construction decision variable. As mentioned in Section 2.1, the network topology design has a discrete form and a continuous form. For instance, the vector $\boldsymbol{\delta}$ in the discrete form can be defined as a decision variable corresponding to adding or removing links or lanes in the network. In this case, the mth element $\boldsymbol{\delta}_m$ of $\boldsymbol{\delta}$ corresponds to the number of lanes that will be added to or subtracted from link[1] m, where $\boldsymbol{\delta}_m$ with a positive value means addition of lanes, and $\boldsymbol{\delta}_m$ with a negative value means removal of lanes. In this paper, we assume that the construction decision variable $\boldsymbol{\delta}_m$ may have a negative value, which means that links or lanes can be blocked in order to improve the performance of the network[2]. Note that the value of $\boldsymbol{\delta}_m$ is constrained by the current number of lanes on link m and the available free space: $\boldsymbol{\delta}_m^{\min} \leqslant \boldsymbol{\delta}_m \leqslant \boldsymbol{\delta}_m^{\max}$. More specifically, the lower bound value $\boldsymbol{\delta}_m^{\min}$ should guarantee that the total number of lanes on link m after construction is non-negative, and the upper bound value $\boldsymbol{\delta}_m^{\max}$ should guarantee that the total number of lanes on link m after construction does not exceed the physical limitations. Similarly, for the continuous form, the vector $\boldsymbol{\delta}$ corresponds to e.g. expansion or reduction of link capacity, and $\boldsymbol{\delta}_m$ corresponds to the width of lanes that will be added to or subtracted from link m.

Changing the network topology will definitely influence the traffic flow patterns, which means that the traffic state x (usually traffic flow, density, and speed) depends on the construction decision variable $\boldsymbol{\delta}$. Thus, traffic evolution can be described by a difference equation as follows:

$$x(k+1) = f(x(k), u(k), d(k), \boldsymbol{\delta}) \,, \tag{1}$$

[1] Link m can initially be a virtual link with 0 lanes. In that case, a new link m is considered to be built with $\boldsymbol{\delta}_m$ lanes ($\boldsymbol{\delta}_m \geqslant 0$).

[2] Braess et al. (2005) illustrate that adding new links may in some cases deteriorate the performance of the traffic network.

where f is the traffic update function, $x(k) \in \mathbb{R}^{n_x}$ denotes the traffic state vector at simulation step k, $u(k) \in \mathbb{R}^{n_u}$ denotes the control inputs vector at simulation step k, and $d(k) \in \mathbb{R}^{n_d}$ denotes the disturbance (typically, the traffic demand) at simulation step k.

4.2. Parameterized traffic control

A parameterized traffic control law is generally formulated as:

$$u_{\mathrm{c}}(k_{\mathrm{c}}) = h(x(Mk_{\mathrm{c}}), d(Mk_{\mathrm{c}}), \boldsymbol{\theta}) \ , \tag{2}$$

where h denotes the control law, $u_{\mathrm{c}}(k_{\mathrm{c}}) \in \mathbb{R}^{n_u}$ denotes the control inputs vector at control step k_{c}, and $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ contains all the parameters of the traffic controllers. The integer M is calculated by $M = T_{\mathrm{c}}/T$, with T_{c} the control interval and T the simulation interval. For the sake of simplicity, we assume that T is an integer divisor of T_{c}. We make an explicit difference between the control interval T_{c} and the simulation interval T, because traffic evolution is a comparatively fast process, while the control actions are not necessarily updated as fast as the traffic evolution. Therefore, the relationship between the control inputs $u_{\mathrm{c}}(k_{\mathrm{c}})$ and $u(k)$ is captured as $u(k) = u_{\mathrm{c}}(\lfloor k/M \rfloor)$, where the operator $\lfloor \cdot \rfloor$ denotes the largest integer less than or equal to the function argument.

Two examples of the parameterized traffic control measures for the discrete-time, macroscopic model are formulated in this section, and will be used for the case study in Section 6. Note that one can design a parameterized control law for any traffic model, or for any type of control measure, so the formulation of the control law does not necessarily have to follow the structures to be presented here.

- A possible control law for ramp metering originates from ALINEA[3] (Papageorgiou et al., 1991):

$$r_o(k_{\mathrm{c}}) = \min\Big(\max\Big(r_o(k_{\mathrm{c}} - 1) + \boldsymbol{\theta}_o^{\mathrm{r}} \frac{\rho_m^{\mathrm{crit}} - \rho_{m,1}(Mk_{\mathrm{c}})}{\rho_m^{\mathrm{crit}}}, 0 \Big), 1 \Big) \ , \tag{3}$$

  where $r_o(k_{\mathrm{c}})$ denotes the ramp metering rate at the origin o connected to link m at control step k_{c}, $\rho_{m,1}(Mk_{\mathrm{c}})$ denotes the density of the first segment of link m at the simulation step $k = Mk_{\mathrm{c}}$, ρ_m^{crit} denotes the critical density on link m, and $\boldsymbol{\theta}_o^{\mathrm{r}}$ is the metering parameter to be designed. A high value of $\boldsymbol{\theta}_o^{\mathrm{r}}$ means that the ramp metering rate is more sensitive to the change in the traffic density. The functions $\max(\cdot)$ and $\min(\cdot)$ are used to bound the ramp metering rate between 0 and 1.

- A possible control law for variable speed limits is formulated as follows (Zegeye et al., 2012):

$$\begin{aligned} v_{m,i}^{\mathrm{lim}}(k_{\mathrm{c}}) = \min\Big(\max\Big(&\boldsymbol{\theta}_{m,0}^{\mathrm{v}} v_{m,i}^{\mathrm{lim}}(k_{\mathrm{c}} - 1) + \boldsymbol{\theta}_{m,1}^{\mathrm{v}} \frac{v_{m,i+1}(Mk_{\mathrm{c}}) - v_{m,i}(Mk_{\mathrm{c}})}{v_{m,i+1}(Mk_{\mathrm{c}}) + \kappa_{m,v}} \\ &+ \boldsymbol{\theta}_{m,2}^{\mathrm{v}} \frac{\rho_{m,i+1}(Mk_{\mathrm{c}}) - \rho_{m,i}(Mk_{\mathrm{c}})}{\rho_{m,i+1}(Mk_{\mathrm{c}}) + \kappa_{m,\rho}}, v_m^{\mathrm{min}} \Big), v_m^{\mathrm{free}} \Big) \ , \end{aligned} \tag{4}$$

[3] In ALINEA, the occupancy, which is defined as the fraction of time that vehicles are detected by the sensors, is used to determine the ramp metering rate, while in our approach, the traffic density is used in the control law instead. The reason is that the traffic density is usually used as a state variable in many macroscopic traffic models; however, it is not easy to be directly measured by the sensors, so the occupancy is often used in practice. Note that there is an approximate relationship between the traffic density and the occupancy (Athol, 1965): occupancy = density × average vehicle length.

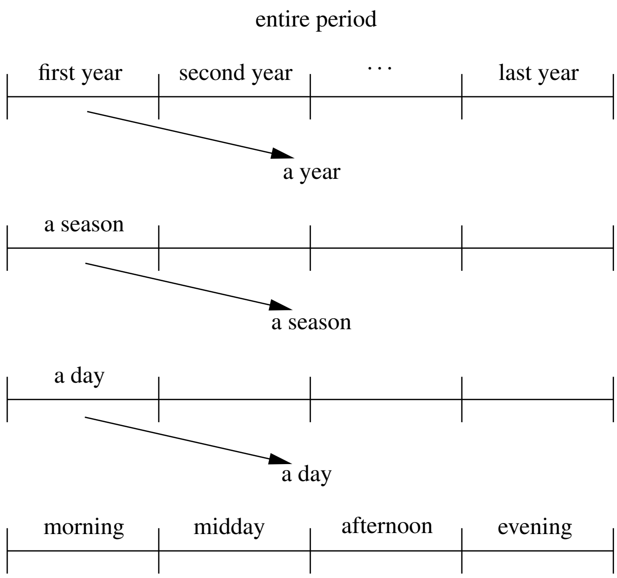


Figure 3: Sub-periods for quasi-dynamic parameterized traffic control

where $v^{\mathrm{lim}}_{m,i}(k_{\mathrm{c}})$ denotes the maximum allowed speed of segment i on link m at control step k_{c}, and $\kappa_{m,v}$ and $\kappa_{m,\rho}$ are parameters preventing denominators from becoming zero. The constants $\boldsymbol{\theta}^{\mathrm{v}}_{m,0}$, $\boldsymbol{\theta}^{\mathrm{v}}_{m,1}$, and $\boldsymbol{\theta}^{\mathrm{v}}_{m,2}$ are the target parameters to be designed. The maximum allowed variable speed $v^{\mathrm{lim}}_{m,i}(k_{\mathrm{c}})$ is bounded by the free flow speed v^{free}_{m} and the minimum speed v^{min}_{m}.

In principle, the parameter vector $\boldsymbol{\theta}$ should be constant for a relatively long time, i.e., at the same time scale as the topology design. However, simple static traffic control using the same parameters over a long period may be inadequate for modern traffic systems. In order to improve the performance of the traffic controllers, we consider a so-called quasi-dynamic parameterized traffic control approach. As shown in Figure 3, the entire design period is divided into years, a year can be divided into seasons, a season can be divided into days, and a day can be even further divided into different time slots, e.g., morning rush hours, midday non-rush hours, afternoon rush hours, and evening non-rush hours. We assume that driver behavior and traffic situations such as weather condition change in different seasons, so a season is considered as the basic unit for a set of parameters in the quasi-dynamic parameterized control. For one day, the parameters in different time slots have different values, while for the same time slot, the parameters in different days have the same value. In this way, we only need to optimize the parameters for one day, but can apply them for a season. In such a way, for complicated traffic situations, the parameters can be defined in a much more refined way than in the static traffic control.

4.3. Traffic models

The co-design problem uses a model-based optimization approach to determine the topology design decisions and the parameters of the traffic control laws. The traffic model used in this approach consists of two parts: a traffic flow model, and a route choice model. In this paper, as an example, we choose the METANET model (Messmer and Papageorgiou, 1990) as the traffic flow model, the multinomial logit model (Sheffi, 1985) as the route choice model. However, it is important to note that our approach is generic, so any other traffic flow model and route choice model can replace the ones used here.

As we have a network with multiple destinations involving route choice, we select the destination-dependent version of METANET model. The destination-dependent METANET model has the following characteristics:

- The traffic network is divided into links corresponding to homogeneous freeway stretches, and each link m is divided into N_m segments of equal length L_m (typically 500-1000 m);
- The traffic network has three different types of nodes, origin $o \in O$, destination $d \in D$, and intermediate node $n \in N$, with O, D and N the set of origins, destinations and intermediate nodes respectively;
- Each segment i of link m is characterized by density $\rho_{m,i,d}(k)$ (veh/km/lane), mean speed $v_{m,i}(k)$ (km/h), outflow $q_{m,i}(k)$ (veh/h) at simulation step k, and number of lanes λ_m;
- Each origin is described by a traffic waiting queue with length $w_{o,d}(k)$ (veh);
- At each segment, the composition rate $\gamma_{m,i,d}(k)$ denotes the fraction of traffic flow to destination d on segment i of link m at simulation step k;
- At each intermediate node, the splitting rate $\beta_{n,m,d}(k)$ expresses the fraction of the total flow with destination d leaving node n via outgoing link m at simulation step k.

The main equations of the METANET model can be found in the appendix.

The route choice behavior of drivers in the METANET model is described by the splitting rate $\beta_{n,m,d}(k)$. In our case, the splitting rate $\beta_{n,m,d}(k)$ is determined by the travel time according to the logit model:

$$\beta_{n,m,d}(k) = \frac{e^{-\xi_{n,d} t_{n,m,d}(k)}}{\sum_{m' \in O_n} e^{-\xi_{n,d} t_{n,m',d}(k)}}\,, \tag{5}$$

where $t_{n,m,d}(k)$ denotes the predicted travel time at node n to destination d via link m at simulation step k, and O_n denotes the set of outgoing links from node n. The predicted travel time $t_{n,m,d}(k)$ can be communicated to the drivers via variable message signs, on-board devices, or on-line traffic information (e.g., radio or other resources). The parameter $\xi_{n,d} > 0$ reflects how drivers react on a travel time difference between different routes to destination d at node n. The higher $\xi_{n,d}$, the less travel time difference is needed to convince drivers to choose the fastest route.

4.4. Performance criteria

The objective of the co-design problem is two-fold:

- To improve the traffic flows in the network;
- To reduce the construction and maintenance cost.

4.4.1. Flow performance criteria

As an illustration, let us interpret improving the traffic flows as minimizing both the total time spent (TTS) and total distance spent (TDS) by all the vehicles in the network. We define a day as the representative basic design period. The TTS and the TDS for the given day is formulated by a monetary valuation, and can be easily calculated for METANET as follows:

$$J_{\mathrm{TTS}} = \sum_{k\in K} \sum_{d\in D} \Big(\sum_{m\in M} \sum_{i\in I_m} \alpha^{\mathrm{t}} \rho_{m,i,d}(k) L_m \lambda_m T + \sum_{o\in O} \alpha^{\mathrm{w}} w_{o,d}(k) T \Big) + J_{\mathrm{TTS,endpoint}} \,, \tag{6}$$

$$J_{\mathrm{TDS}} = \sum_{k\in K} \sum_{m\in M} \sum_{i\in I_m} \alpha^{\mathrm{d}} q_{m,i}(k) T L_m + J_{\mathrm{TDS,endpoint}} \,, \tag{7}$$

with α^{t} (\$ /h) the monetary cost per unit travel time, α^{w} (\$ /h) the monetary cost per unit waiting time, α^{d} (\$ /km) the monetary cost per unit distance, $w_{o,d}(k)$ the number of vehicles to destination d waiting on origin o at simulation step k, M the set of links of the network, I_m the set of segments on link m, O the set of origins, K the set of simulation steps for the given day, and $J_{\mathrm{TTS,endpoint}}$ and $J_{\mathrm{TDS,endpoint}}$ are end point penalties. The factor $\rho_{m,i,d}(k) L_m \lambda_m$ in (6) indicates the number of vehicles with destination d in segment i of link m, and hence multiplied by the time interval T this gives the time spent by the vehicles in the corresponding segment. Similarly, the term $q_{m,i}(k)T$ in (7) represents the number of vehicles leaving segment i of link m, and multiplied by the length of segment L_m this gives the distance traveled by the vehicles. Note that because we consider a representative day as the basic design period, it is possible that some vehicles may be still traveling in the network, or even waiting at origins at the end of the day. These vehicles will eventually reach their destination, and hence we should also take the travel time cost and travel distance cost spent after the end of the representative day into account, and add them to J_{TTS} and J_{TDS} respectively. This part of the cost is called an end point penalty (Maciejowski, 2002). The end point penalties $J_{\mathrm{TTS,endpoint}}$ and $J_{\mathrm{TDS,endpoint}}$ can be computed in several ways, e.g.,

1. We estimate the TTS and the TDS spent by all the vehicles that are still in the network or at the origins after the end of the representative day:

$$J_{\mathrm{TTS,endpoint}} = \alpha^{\mathrm{t}} \sum_{d\in D} \Big(\sum_{m\in M} \sum_{i\in I_m} \rho_{m,i,d}(k_{\mathrm{end}}) L_m \lambda_m \tau_{m,i,d} + \sum_{o\in O} w_{o,d}(k_{\mathrm{end}}) \tau_{o,d} \Big) \,, \tag{8}$$

$$J_{\mathrm{TDS,endpoint}} = \alpha^{\mathrm{d}} \sum_{d\in D} \Big(\sum_{m\in M} \sum_{i\in I_m} \gamma_{m,i,d}(k_{\mathrm{end}}) q_{m,i}(k_{\mathrm{end}}) T \ell_{m,i,d} + \sum_{o\in O} w_{o,d}(k_{\mathrm{end}}) \ell_{o,d} \Big) \,, \tag{9}$$

   with $\tau_{m,i,d}$ and $\tau_{o,d}$ the typical travel times that a vehicle in the segment i of link m and at origin o needs to reach destination d, $\ell_{m,i,d}$ and $\ell_{o,d}$ the typical travel distances that a vehicle in the segment i of link m and at origin o needs to reach destination d, and $k_{\mathrm{end}} \in K$ the last simulation step of K. The typical travel time and distance can be calculated based on historical data.

2. We use the same expressions of (8) and (9) to calculate the end point penalties, but the typical travel time $\tau_{m,i,d}(k_{\mathrm{end}})$ and $\tau_{o,d}(k_{\mathrm{end}})$ and the typical travel distance $\ell_{m,i,d}$ and $\ell_{o,d}$ are determined according to the traffic state on the fastest or shortest route at simulation step

k_{end}:

$$\tau_{m,i,d}(k_{\text{end}}) = \sum_{j=i+1}^{N_m} \frac{L_m}{v_{m,j}(k_{\text{end}})} + \sum_{l \in r_{m,d}} \sum_{j=1}^{N_l} \frac{L_l}{v_{l,j}(k_{\text{end}})} , \tag{10}$$

$$\tau_{o,d}(k_{\text{end}}) = \sum_{l \in r_{o,d}} \sum_{j=1}^{N_l} \frac{L_l}{v_{l,j}(k_{\text{end}})} , \tag{11}$$

$$\ell_{m,i,d} = \sum_{j=i+1}^{N_m} L_m + \sum_{l \in r_{m,d}} \sum_{j=1}^{N_l} L_l , \tag{12}$$

$$\ell_{o,d} = \sum_{l \in r_{o,d}} \sum_{j=1}^{N_l} L_l , \tag{13}$$

where $r_{m,d}$ denotes the fastest or shortest route from the end of link m to destination d, and $r_{o,d}$ denotes the fastest or shortest route from origin o to destination d.

3. We determine K shortest or fastest loop-less routes (Yen, 1971; Katoh et al., 1982; Hadjiconstantinou and Christofides, 1999) from any link end and any origin to any destination — provided that some vehicles are still traveling between that link or origin and that destination at the end of the representative day — and then those vehicles are distributed over these K shortest routes. The end point penalties are then computed by summing the total travel time cost and the total travel distance cost for all the vehicles remaining in the network at simulation step k_{end}.
4. We keep simulating the traffic network until all the vehicles have left it, while setting the demand to zero after the end of the representative day. The end point penalties are directly yielded according to the simulation results.

One should choose one of these four approaches w.r.t finding a balanced trade-off between the accuracy and the computation speed when computing the end point penalty.

For each sub-period (e.g., week, season, year) corresponding to the quasi-dynamic approach of Section 4.2 and Figure 3, by computing the TTS and the TDS for those days using (6) and (7), and next multiplying the result with the number of days that of the given sub-period, we can compute yearly monetary TTS and TDS values $J_{\text{TTS},y}$ and $J_{\text{TDS},y}$ for each year y in the full period under consideration.

4.4.2. Construction and maintenance performance criteria

Another objective is to reduce the construction and maintenance cost of the network. A linear construction cost function can be adopted as:

$$J_{\text{CC}} = \sum_{\substack{m \in M \\ \delta_m > 0}} \alpha_m^{\text{c}} N_m L_m \boldsymbol{\delta}_m - \sum_{\substack{m \in M \\ \delta_m < 0}} \alpha_m^{\text{r}} N_m L_m \boldsymbol{\delta}_m , \tag{14}$$

where α_m^{c} (\$ /year) denotes the construction cost per lane per unit length, α_m^{r} (\$ /year) denotes the removal cost per lane per unit length, and N_m denotes the number of segments on link m. The maintenance cost depends on the number of lanes after construction, and the cost for the first year is formulated as:

$$J_{\mathrm{MC}} = \sum_{m \in M} \alpha_m^{\mathrm{m}} N_m L_m (\lambda_m + \boldsymbol{\delta}_m) \,, \tag{15}$$

where α_m^{m} (\$ /year) denotes the maintenance cost per lane per unit length.

4.4.3. Overall performance criteria

The overall objective function can be obtained by summing up all the objective functions. Note that the major difference between the construction cost and the other three costs is that the construction cost is only spent in the first year, while the other three costs should be considered every year, and will be increasing annually due to the inflation effect. We assume that the inflation effect starts at the beginning of every year, so the overall objective function is formulated as:

$$J = J_{\mathrm{CC}} + \sum_{y=1}^{N_{\mathrm{y}}} (1+r)^{y-1} (J_{\mathrm{MC}} + J_{\mathrm{TTS},y} + J_{\mathrm{TDS},y}) \,, \tag{16}$$

where r denotes the yearly inflation rate, and N_{y} denotes the number of years during the entire design period.

4.5. Constraints

Recall that as introduced in Section 4.1 the value of the topology design decision variable $\boldsymbol{\delta}_m$ is constrained by the number of lanes on link m and the available free space: $\boldsymbol{\delta}_m^{\min} \leqslant \boldsymbol{\delta}_m \leqslant \boldsymbol{\delta}_m^{\max}$. Moreover, in order to avoid reducing the region of possible solutions, or causing an infeasible problem, constraints on the parameters of control laws are not directly put on the parameters themselves, but instead a minimum and a maximum value is added to each control signal as shown in (3) and (4). For traffic states, all the equality constraints such as the conversation law of traffic flows have been included in the traffic simulation model, and thus eliminated.

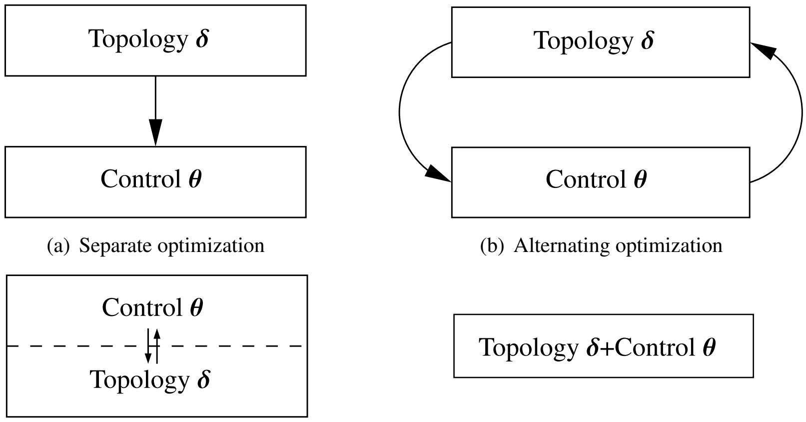
4.6. Overall optimization problem

So far we have introduced the objective functions and the constraints for optimization, together with the topology design methods, traffic control measures, and traffic models. The overall optimization problem can be formulated as:

$$\begin{aligned} \min_{\boldsymbol{\theta},\boldsymbol{\delta}} \quad & J(\boldsymbol{\theta},\boldsymbol{\delta}) \\ \text{subject to} \quad & y(\boldsymbol{\theta},\boldsymbol{\delta}) = 0 \\ & z(\boldsymbol{\theta},\boldsymbol{\delta}) \leqslant 0 \end{aligned} \tag{17}$$

where $J(\cdot)$ includes all the objective functions, $y(\cdot) = 0$ includes all the equality constraints, and $z(\cdot) \leqslant 0$ includes all the inequality constraints. For the sake of compactness in Section 5, the constraints of (17) will be denoted via a constraint set C

$$C = \{(\boldsymbol{\theta},\boldsymbol{\delta}) | y(\boldsymbol{\theta},\boldsymbol{\delta}) = 0 \text{ and } z(\boldsymbol{\theta},\boldsymbol{\delta}) \leqslant 0\} \,. \tag{18}$$



(a) Separate optimization

(b) Alternating optimization

(c) Bi-level optimization

(d) Joint optimization

Figure 4: Schematic representation of four solution frameworks

In addition, we define $C_{\boldsymbol{\theta}}(\boldsymbol{\delta})$ and $C_{\boldsymbol{\delta}}(\boldsymbol{\theta})$ as the sets of feasible $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$ for a given value of $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$ respectively, i.e.,

$$C_{\boldsymbol{\theta}}(\boldsymbol{\delta}) = \{\boldsymbol{\theta} | (\boldsymbol{\theta}, \boldsymbol{\delta}) \in C\} \tag{19}$$

$$C_{\boldsymbol{\delta}}(\boldsymbol{\theta}) = \{\boldsymbol{\delta} | (\boldsymbol{\theta}, \boldsymbol{\delta}) \in C\} \,. \tag{20}$$

5. Solution approaches

5.1. Solution frameworks

We have described an optimization formulation (17) in order to solve the co-design problem of the network topology and traffic control measures. However, optimizing both $\boldsymbol{\theta}$ and $\boldsymbol{\delta}$ will potentially result in a high computational burden. Therefore, the development of an efficient solution approach is required to guarantee successful implementation of the co-design problem.

To solve (17), we discuss four solution frameworks, i.e., separate optimization, iterative optimization, bi-level optimization, and joint optimization (see Figure 4). We introduce each framework in more detail next.

5.1.1. Separate optimization

Separate optimization means that the optimization problem is decomposed into multiple separate subproblems. This framework has been applied to solve e.g. the machining optimization problem (Shin and Joo, 1992), and the mesh optimization problem (Hoppe et al., 1993). In our case, we first determine the optimal topology design decision variable $\boldsymbol{\delta}$ by fixing the parameters of the traffic control law to $\boldsymbol{\theta}_{\text{fix}}$:

$$\boldsymbol{\delta}^* = \arg \min_{\boldsymbol{\delta} \in C_{\boldsymbol{\delta}}(\boldsymbol{\theta}_{\text{fix}})} J(\boldsymbol{\theta}_{\text{fix}}, \boldsymbol{\delta}) \,. \tag{21}$$

The value of $\boldsymbol{\theta}_{\text{fix}}$ can be chosen according to the default settings of the control laws, or based on knowledge from experts. After $\boldsymbol{\delta}^*$ is determined, we then only optimize the parameters $\boldsymbol{\theta}$ of the control laws in the network with the fixed topology:

$$\boldsymbol{\theta}^{*} = \arg \min_{\boldsymbol{\theta} \in C_{\boldsymbol{\theta}}(\boldsymbol{\delta}^{*})} J(\boldsymbol{\theta}, \boldsymbol{\delta}^{*}) \tag{22}$$

This yields the pair $(\boldsymbol{\delta}^{*}, \boldsymbol{\theta}^{*})$ as the approximate solution of the optimization problem (17).

5.1.2. Alternating optimization

A more advanced approach compared to separate optimization is iterative optimization. In iterative optimization, we also solve the optimization problems (21) and (22), but these two problems are solved iteratively instead of only once. The solution $\boldsymbol{\theta}^{*}$ from (22) is fed back to (21) by replacing $\boldsymbol{\theta}_{\text{fix}}$ with $\boldsymbol{\theta}^{*}$, and in this way, a new solution of $\boldsymbol{\delta}^{*}$ can be obtained. Generally, we repeat solving:

$$\boldsymbol{\delta}^{*^{i+1}} = \arg \min_{\boldsymbol{\delta} \in C_{\boldsymbol{\delta}}\left(\boldsymbol{\theta}^{*^{i}}\right)} J\left(\boldsymbol{\theta}^{*^{i}}, \boldsymbol{\delta}\right) \tag{23}$$

$$\boldsymbol{\theta}^{*^{i+1}} = \arg \min_{\boldsymbol{\theta} \in C_{\boldsymbol{\theta}}\left(\boldsymbol{\delta}^{*^{i+1}}\right)} J\left(\boldsymbol{\theta}, \boldsymbol{\delta}^{*^{i+1}}\right) \tag{24}$$

until one of the following stop criteria is satisfied:

1. The maximum number of iteration steps N is reached;
2. The difference of values of $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$ between two consecutive iteration steps are both smaller than some predefined threshold: $\|\boldsymbol{\delta}^{*^{i+1}} - \boldsymbol{\delta}^{*^{i}}\| < n_{\boldsymbol{\delta}}$ and $\|\boldsymbol{\theta}^{*^{i+1}} - \boldsymbol{\theta}^{*^{i}}\| < n_{\boldsymbol{\theta}}$.

More information about the iterative optimization approach can be found in (Bezdek and Hathaway, 2002; Boyd et al., 2011).

5.1.3. Bi-level optimization

Bi-level optimization (Candler and Townsley, 1982) divides the optimization problem (17) into two levels (an inner one and an outer one), where one problem is embedded within another. Generally, the outer optimization task is commonly referred to as the upper-level optimization task, and the inner optimization task is commonly referred to as the lower-level optimization task. These problems involve two kinds of variables, referred to as the upper-level variables and the lower-level variables. In the given problem setting, it is assumed that for any network topology (a given $\boldsymbol{\delta}$), there exists a corresponding optimal parameter $\boldsymbol{\theta}^{*}(\boldsymbol{\delta})$ of the control law. Therefore, in the bi-level optimization, the lower level aims at finding a relationship between $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$, and the upper level aims at determining the network topology.

The relationship between $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$ can be expressed by a function $\boldsymbol{\theta}^{*}(\boldsymbol{\delta})$, which can be obtained from the lower level optimization

$$\boldsymbol{\theta}^{*}(\boldsymbol{\delta}) = \arg \min_{\boldsymbol{\theta} \in C_{\boldsymbol{\theta}}(\boldsymbol{\delta})} J(\boldsymbol{\theta}, \boldsymbol{\delta}) \ . \tag{25}$$

In the upper level, the topology design decision is determined according to

$$\boldsymbol{\delta}^{*} = \arg \min_{\boldsymbol{\delta} \in C_{\boldsymbol{\delta}}} J(\boldsymbol{\theta}^{*}(\boldsymbol{\delta}), \boldsymbol{\delta}) \ , \tag{26}$$

with $C_\delta = \{\boldsymbol{\delta} | \exists \boldsymbol{\theta} \text{ such that } (\boldsymbol{\theta}, \boldsymbol{\delta}) \in C\}$. The optimal parameter is obtained by:

$$\boldsymbol{\theta}^* = \boldsymbol{\theta}^*(\boldsymbol{\delta}^*) \tag{27}$$

5.1.4. Joint optimization

The aforementioned three frameworks deal with the topology design and traffic control measures one after the other. Unlike them, joint optimization solves (17) by considering both $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$ at the same time, which can be obtained by:

$$(\boldsymbol{\theta}^*, \boldsymbol{\delta}^*) = \arg \min_{(\boldsymbol{\theta},\boldsymbol{\delta})\in C} J(\boldsymbol{\theta}, \boldsymbol{\delta}) \tag{28}$$

5.1.5. Discussion

One should consider the complexity of the co-design problem when choosing the solution framework. The complexity of the problem changes when the design settings are different. For instance, choosing a week as the basic design period, in which the expected traffic demand and traffic situations can be distinguished between weekdays and weekends, can yield more refined design results than choosing a day as the basic design period. However, the computational burden of simulation will be also higher by using a longer basic design period. Among the aforementioned solution frameworks, the separate optimization approach is the easiest approach to implement. The iterative optimization approach will in general achieve a better performance than the separate optimization approach, but the computation speed will become slower because of the iterative procedure. Cantarella et al. (2006) present the iterative optimization approach to solve a co-design problem with network topology and traffic signal setting. To the best knowledge of the authors, the co-design problem by using the bi-level optimization approach or the joint optimization approach has not yet been investigated in the literature . In these two approaches, the topology design and the traffic control can interact with each other well, so the performance is expected to be improved significantly. However, the computation time will also increase highly. Therefore, an appropriate solution framework should be selected to provide a balance between the performance and the computation speed according to the design requirements.

5.2. Optimization algorithms

In general, the co-design problem will result in a non-linear, non-convex optimization problem. To tackle such a problem, different optimization algorithms can be applied.

- For real-valued problems, i.e., when the topology design has a continuous form and the parameters of the control law have continuous values, we can use multi-start sequential quadratic programming (Boggs and Tolle, 1995), pattern search (Hooke and Jeeves, 1961), simulated annealing (Kirkpatrick et al., 1983), genetic algorithms (Bäck, 1996), and so on;
- For mixed-integer problems, i.e., when the topology design has a discrete form or the parameters of the control law have discrete values, genetic algorithms and simulated annealing are also suitable. Moreover, other mixed-integer nonlinear programming approaches (Belotti et al., 2013) such as branch-and-bound algorithms (Lawler and Wood, 1966), branch-and-cut algorithms (Tawarmalani and Sahinidis, 2005), outer approximation method (Duran and Grossmann, 1986), and Benders decomposition method (Geoffrion, 1972) can be used as well.

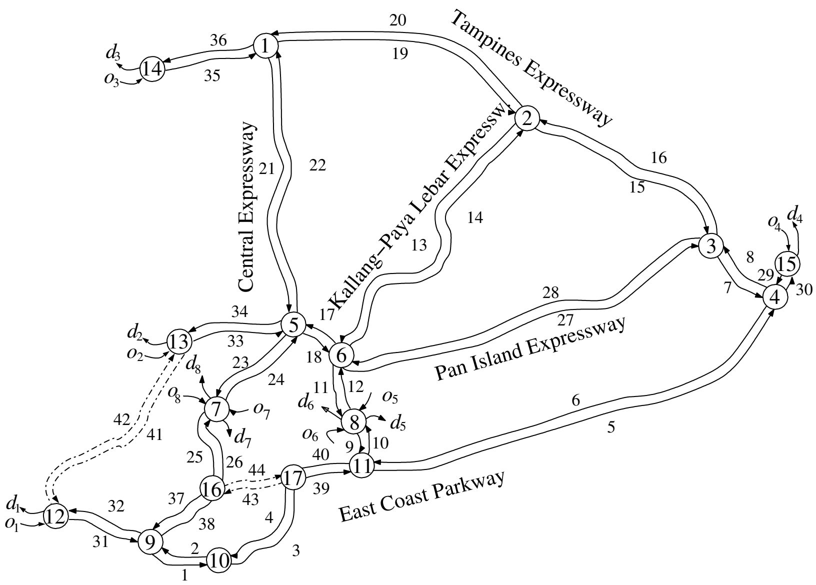


Figure 5: The central and eastern parts of the Singapore expressway network

6. Case study

6.1. Set-up

We illustrate the four proposed solution frameworks to the co-design problem introduced in Section 5.1 in a simulation of the central and eastern parts of the Singapore expressway network (Figure 5), which is one of the busiest areas in Singapore, including the central business district and the international airport. This area contains 40 one-way highway links (with the parameters of each link presented in Table 1), 8 origins (o_1–o_8), and 8 destinations (d_1–d_8).

The topology design decision is to determine whether new links should be constructed or existing links should be removed, or whether the numbers of lanes on the existing links should be changed. According to the simulation scenario (see Section 6.3), we assume that new links 41 and 42 can be built between nodes 12 and 13, and new links 43 and 44 can be built between nodes 16 and 17, as shown by dash lines in Figure 5. The upper bound on the number of lanes for each new link is set as 2. Moreover, we may change the numbers of lanes on links 9, 17, 23, 25, 32, and 33, which are main roads around the central business district. The upper bound of the number of lanes for each existing link is set as 4. As traffic control measures, ramp metering installations are put at origins o_2, o_4, o_5, and o_7, and variable speed limits are installed on link 32, as well as link 41 if it is built. Only the segments on the second half of link 32 and 41 are controlled by the dynamic speed limits, which is similar to the settings considered by Hegyi et al. (2005b). Eventually, we have a co-optimization problem with 10 integer optimization variables and 10 continuous optimization variables.

6.2. Optimization and model parameters

The cost of construction of highway projects could be affected by several factors, such as terrain type (mountainous or flat), material type (concrete or asphalt), development type (rural

Table 1: Parameters of links in the Singapore expressway network

Link index	Length (km)	No. of lanes	Nodes	Capacity (veh/h)
1, 2	3.0	3	9, 10	6000
3, 4	3.5	4	10, 17	8000
5, 6	13.0	4	11, 4	8000
7, 8	2.0	4	3, 4	8000
9, 10	1.0	3	8, 11	6000
11, 12	2.0	3	6, 8	6000
13, 14	8.0	3	2, 6	6000
15, 16	6.5	3	2, 3	6000
17, 18	2.0	4	6, 5	8000
19, 20	7.0	3	1, 2	6000
21, 22	7.5	2	1, 5	4000
23, 24	3.5	2	5, 7	4000
25, 26	2.5	2	7, 16	4000
27, 28	11.0	4	6, 3	8000
29, 30	1.0	4	15, 4	8000
31, 32	3.0	3	12, 9	6000
33, 34	3.0	4	13, 5	8000
35, 36	3.0	3	14, 1	6000
37, 38	2.0	2	9, 16	4000
39, 40	1.0	4	11, 17	8000

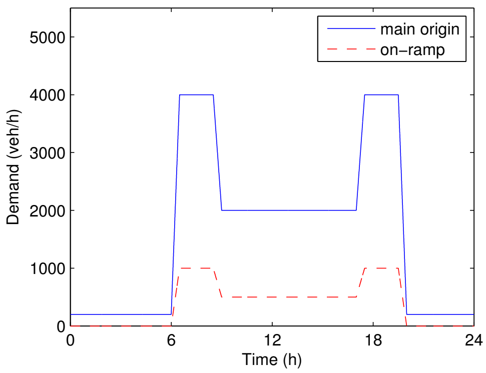
or urban), and so on. According to a report by the U.S. General Accounting Office (2003), the unit cost of building a stretch of highway ranged from about \$2.5 million per km to \$16 million per km in 25 U.S. states in 2002. In this case study, we set the construction cost $\alpha_m^{\mathrm{c}} = \$10$ million per lane per km, and the removal cost $\alpha_m^{\mathrm{r}} = \$5$ million per lane per km. The maintenance cost for the first year is $\alpha_m^{\mathrm{m}} = \$1$ million per lane per km, and it will increase every year at the rate of $r = 0.04$. The total length of the entire design period is set as $N_{\mathrm{y}} = 20$ years. For other parameters, we assume that the travel time cost is equal to the waiting time cost, defined as $\alpha^{\mathrm{t}} = \alpha^{\mathrm{w}} = \$10$ per h per vehicle, and the travel distance cost is set as $\alpha^{\mathrm{d}} = \$1$ per km per vehicle.

The destination-dependent METANET model (see Appendix) is used to simulate the traffic flow evolution, and we use the traffic control laws (3) and (4) in this case study. The model parameters are defined as follows (Hegyi et al., 2005a): simulation time step length $T = 10$ s, free flow speed $v_m^{\mathrm{free}} = 120$ km/h, lower bound of speed limit $v_m^{\mathrm{min}} = 50$ km/h, critical density $\rho_m^{\mathrm{crit}} = 33.5$ veh/km/lane, maximum density $\rho_m^{\mathrm{max}} = 180$ veh/km/lane, flow capacity $q_{\mathrm{cap}} = 2000$ veh/h/lane. Other parameters can be found in (Kotsialos et al., 2002a) or (Hegyi et al., 2005a).

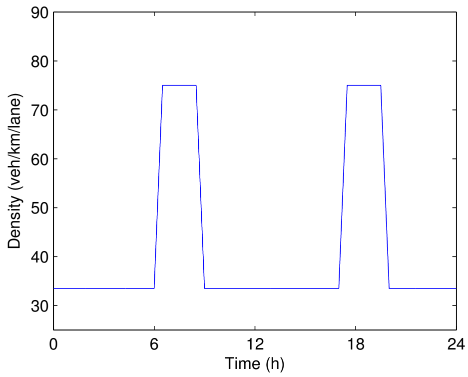
6.3. Scenario

We consider four different traffic inflows in the network (see Figure 6(a)) from origins o_2, o_4, o_5, and o_7, and all of them have the same destination d_1.

- For traffic flows from o_2 and o_4, we define a high traffic demand profile: it starts with a flow of 200 veh/h at 12.00 a.m., reaches the first peak of 4000 veh/h at 6.30 a.m., gradually



(a) Demand profile



(b) Downstream density profile

Figure 6: Traffic scenario considered in the simulation experiment

decreases to 2000 veh/h at 10.00 a.m., reaches the second peak of 4000 veh/h at 5.30 p.m., decreases to 200 veh/h at 9.00 p.m., and maintains this level until midnight;

- For traffic flows from o_5 and o_7, we define a low traffic demand profile: it starts with a flow of 0 veh/h at 12.00 a.m., reaches the first peak of 1000 veh/h at 6.30 a.m., decreases to 100 veh/h at 10.00 a.m., reaches the second peak of 1000 veh/h at 5.30 p.m., decreases to 0 veh/h at 9.00 p.m., and maintains this level until midnight.

In order to create traffic congestion at the peak hours, we add a large pulse with a maximum value of 75 veh/km/lane to the downstream density of the network at each peak-hour period, as shown in Figure 6(b). These pulses can generate a back-propagating wave and make the traffic in the network more busy.

6.4. Simulation results

We use the four solution frameworks to solve the co-design problem for the Singapore expressway network, and compare the results. In order to prevent that the results from different solution frameworks would be fully determined by the optimization algorithm, two optimization algorithms are used to benchmark all the solution frameworks: one is a genetic algorithm implemented in the `ga` function of the Matlab Global Optimization Toolbox, and the other is an outer approximation branch-and-bound algorithm implemented in the `minlp` function of the TOMLAB Mixed-Integer Nonlinear Programming toolbox for Matlab. The default parameters settings of the `ga` function are used in this case study. More information about parameter tuning of genetic algorithms can be found in Eiben et al. (1999); Lobo et al. (2007).

The results of the four solution frameworks by using `ga` and `minlp` are displayed in Tables 2 and 3, and for comparison, we plot the total cost of the four solution frameworks by using the two optimization algorithms in Figure 6.4. In this figure, we can see that for each solution framework the total cost is almost the same by using the `ga` and `minlp` functions, except that in the separate optimization framework, the total cost by using the `minlp` function is a little higher than the one by using the `ga` function. Therefore, we can conclude that the simulation results are mainly determined by the solution framework, not by the optimization algorithms. Moreover,

Table 2: Simulation results by using `ga`

Solution framework	Total cost J (\$)	construction decision δ		control parameters θ	
Separate optimization	$4.6431 \cdot 10^{10}$	$\delta_9 = -2$	$\delta_{25} = -1$	$\theta^r_2 = 0.61$	$\theta^v_{31,1} = 2.36$
		$\delta_{12} = -2$	$\delta_{41} = 2$	$\theta^r_4 = 0.73$	$\theta^v_{31,2} = 0.04$
		$\delta_{17} = -4$	$\delta_{42} = 0$	$\theta^r_5 = 0.10$	$\theta^v_{41,0} = 1.92$
		$\delta_{23} = -2$	$\delta_{43} = 2$	$\theta^r_7 = 1.65$	$\theta^v_{41,1} = 1.78$
		$\delta_{24} = -2$	$\delta_{44} = 0$	$\theta^v_{32,0} = 1.37$	$\theta^v_{41,2} = 2.80$
Alternating optimization	$3.6430 \cdot 10^{10}$	$\delta_9 = -1$	$\delta_{25} = -1$	$\theta^r_2 = 13.72$	$\theta^v_{31,1} = 337.81$
		$\delta_{12} = -3$	$\delta_{41} = 2$	$\theta^r_4 = 12.75$	$\theta^v_{31,2} = 1127.17$
		$\delta_{17} = -3$	$\delta_{42} = 0$	$\theta^r_5 = 18.62$	$\theta^v_{41,0} = 1.62$
		$\delta_{23} = -2$	$\delta_{43} = 2$	$\theta^r_7 = 8.99$	$\theta^v_{41,1} = 104.37$
		$\delta_{24} = -2$	$\delta_{44} = 0$	$\theta^v_{32,0} = 1.49$	$\theta^v_{41,2} = 2311.53$
Bi-level optimization	$3.6428 \cdot 10^{10}$	$\delta_9 = -1$	$\delta_{25} = -1$	$\theta^r_2 = 13.23$	$\theta^v_{31,1} = 917.19$
		$\delta_{12} = -3$	$\delta_{41} = 2$	$\theta^r_4 = 11.95$	$\theta^v_{31,2} = 760.89$
		$\delta_{17} = -3$	$\delta_{42} = 0$	$\theta^r_5 = 8.17$	$\theta^v_{41,0} = 1.76$
		$\delta_{23} = -2$	$\delta_{43} = 2$	$\theta^r_7 = 14.50$	$\theta^v_{41,1} = 226.12$
		$\delta_{24} = -2$	$\delta_{44} = 0$	$\theta^v_{32,0} = 1.55$	$\theta^v_{41,2} = 2858.39$
Joint optimization	$3.6399 \cdot 10^{10}$	$\delta_9 = -3$	$\delta_{25} = -1$	$\theta^r_2 = 17.20$	$\theta^v_{32,1} = 315.06$
		$\delta_{12} = -2$	$\delta_{41} = 0$	$\theta^r_4 = 13.21$	$\theta^v_{32,2} = 264.98$
		$\delta_{17} = -4$	$\delta_{42} = 0$	$\theta^r_5 = 10.94$	$\theta^v_{41,0} = 0.73$
		$\delta_{23} = -2$	$\delta_{43} = 0$	$\theta^r_7 = 10.53$	$\theta^v_{41,1} = 375.66$
		$\delta_{24} = -2$	$\delta_{44} = 1$	$\theta^v_{32,0} = 1.19$	$\theta^v_{41,2} = 2677.94$

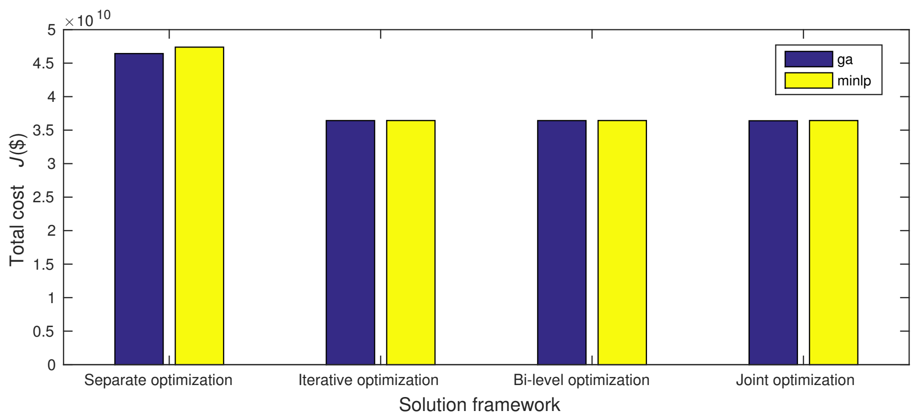


Figure 7: Comparison of total cost of four solution frameworks by using the `ga` and `minlp` functions

Table 3: Simulation results by using `minlp`

Solution framework	Total cost J ($)	construction decision δ		control parameters θ	
Separate optimization	$4.739 \cdot 10^{10}$	$\boldsymbol{\delta}_9 = -2$	$\boldsymbol{\delta}_{25} = -1$	$\boldsymbol{\theta}^{\mathrm{r}}_2 = 1.97$	$\boldsymbol{\theta}^{\mathrm{v}}_{31,1} = 0.06$
		$\boldsymbol{\delta}_{12} = -2$	$\boldsymbol{\delta}_{41} = 2$	$\boldsymbol{\theta}^{\mathrm{r}}_4 = 0.73$	$\boldsymbol{\theta}^{\mathrm{v}}_{31,2} = 0.91$
		$\boldsymbol{\delta}_{17} = -4$	$\boldsymbol{\delta}_{42} = 0$	$\boldsymbol{\theta}^{\mathrm{r}}_5 = 0.01$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,0} = 1.12$
		$\boldsymbol{\delta}_{23} = -2$	$\boldsymbol{\delta}_{43} = 2$	$\boldsymbol{\theta}^{\mathrm{r}}_7 = 0.35$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,1} = 1.77$
		$\boldsymbol{\delta}_{24} = -2$	$\boldsymbol{\delta}_{44} = 0$	$\boldsymbol{\theta}^{\mathrm{v}}_{32,0} = 1.47$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,2} = 0.86$
Alternating optimization	$3.6435 \cdot 10^{10}$	$\boldsymbol{\delta}_9 = -1$	$\boldsymbol{\delta}_{25} = -1$	$\boldsymbol{\theta}^{\mathrm{r}}_2 = 12.09$	$\boldsymbol{\theta}^{\mathrm{v}}_{31,1} = 144.35$
		$\boldsymbol{\delta}_{12} = -3$	$\boldsymbol{\delta}_{41} = 2$	$\boldsymbol{\theta}^{\mathrm{r}}_4 = 13.95$	$\boldsymbol{\theta}^{\mathrm{v}}_{31,2} = 617.42$
		$\boldsymbol{\delta}_{17} = -3$	$\boldsymbol{\delta}_{42} = 0$	$\boldsymbol{\theta}^{\mathrm{r}}_5 = 10.62$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,0} = 1.30$
		$\boldsymbol{\delta}_{23} = -2$	$\boldsymbol{\delta}_{43} = 2$	$\boldsymbol{\theta}^{\mathrm{r}}_7 = 14.03$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,1} = 215.63$
		$\boldsymbol{\delta}_{24} = -2$	$\boldsymbol{\delta}_{44} = 0$	$\boldsymbol{\theta}^{\mathrm{v}}_{32,0} = 1.71$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,2} = 1848.33$
Bi-level optimization	$3.6435 \cdot 10^{10}$	$\boldsymbol{\delta}_9 = -1$	$\boldsymbol{\delta}_{25} = -1$	$\boldsymbol{\theta}^{\mathrm{r}}_2 = 13.08$	$\boldsymbol{\theta}^{\mathrm{v}}_{31,1} = 285.77$
		$\boldsymbol{\delta}_{12} = -3$	$\boldsymbol{\delta}_{41} = 2$	$\boldsymbol{\theta}^{\mathrm{r}}_4 = 8.86$	$\boldsymbol{\theta}^{\mathrm{v}}_{31,2} = 1230.33$
		$\boldsymbol{\delta}_{17} = -3$	$\boldsymbol{\delta}_{42} = 0$	$\boldsymbol{\theta}^{\mathrm{r}}_5 = 10.62$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,0} = 1.49$
		$\boldsymbol{\delta}_{23} = -2$	$\boldsymbol{\delta}_{43} = 2$	$\boldsymbol{\theta}^{\mathrm{r}}_7 = 16.01$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,1} = 293.56$
		$\boldsymbol{\delta}_{24} = -2$	$\boldsymbol{\delta}_{44} = 0$	$\boldsymbol{\theta}^{\mathrm{v}}_{32,0} = 1.03$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,2} = 1474.57$
Joint optimization	$3.6435 \cdot 10^{10}$	$\boldsymbol{\delta}_9 = -1$	$\boldsymbol{\delta}_{25} = -1$	$\boldsymbol{\theta}^{\mathrm{r}}_2 = 12.59$	$\boldsymbol{\theta}^{\mathrm{v}}_{31,1} = 197.78$
		$\boldsymbol{\delta}_{12} = -3$	$\boldsymbol{\delta}_{41} = 2$	$\boldsymbol{\theta}^{\mathrm{r}}_4 = 17.63$	$\boldsymbol{\theta}^{\mathrm{v}}_{31,2} = 761.87$
		$\boldsymbol{\delta}_{17} = -3$	$\boldsymbol{\delta}_{42} = 0$	$\boldsymbol{\theta}^{\mathrm{r}}_5 = 13.46$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,0} = 1.34$
		$\boldsymbol{\delta}_{23} = -2$	$\boldsymbol{\delta}_{43} = 2$	$\boldsymbol{\theta}^{\mathrm{r}}_7 = 13.21$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,1} = 323.92$
		$\boldsymbol{\delta}_{24} = -2$	$\boldsymbol{\delta}_{44} = 0$	$\boldsymbol{\theta}^{\mathrm{v}}_{32,0} = 1.23$	$\boldsymbol{\theta}^{\mathrm{v}}_{41,2} = 1273.11$

it is clear to show that the separate optimization framework results in a much higher total cost than the other three frameworks, which means that the separate optimization framework cannot guarantee the optimal solution of the co-optimization problem. The other three frameworks yield almost the same solution in this case study, especially for the total cost and for the topology design decisions. In Tables 2 and 3, for the construction decision $\boldsymbol{\delta}$, the subscript of each variable corresponds to the index of the link, and the value of each variable indicates the modification of the numbers of lanes on the link, i.e., positive values mean adding lanes, negative values mean removing lanes, and zero values mean no change. For the control parameter $\boldsymbol{\theta}$, the superscript 'r' means that the variable is a parameter for the on-ramp control law, and the superscript 'v' means that the variable is a parameter for the variable speed limits control law. Since the variable speed limits control law has three parameters, we use 0, 1, and 2 in the subscript to distinguish these three parameters. In Table 2, the joint optimization yields different topology design decisions than the iterative optimization and the bi-level optimization, and the total cost is also slightly lower than the one obtained by the other two frameworks. In Table 3, the iterative optimization, the bi-level optimization and the joint optimization yield the same topology design decisions, but different control parameters. However, the values of the total costs of these three solution frameworks are identical for the first five digits. Therefore, in this scenario, the topology design has a more dominant impact on the performance of the traffic network than the traffic control.

7. Conclusions and Future Work

In this paper, we have introduced a co-design method that jointly optimizes the network topology and traffic control parameters. We have proposed four different solution frameworks for such a co-design problem, namely separate optimization, iterative optimization, bi-level optimization, and joint optimization. One should choose the appropriate framework according to the computational complexity requirements. Generally, the separate optimization is the easiest and fastest approach, but the optimal solution usually cannot be guaranteed. The iterative optimization and the bi-level optimization can achieve a better performance than the separate optimization, but their computation speed will decrease due to the increasing computation complexity. The joint optimization has the highest computation complexity, because it usually needs to deal with a nonlinear mixed-integer optimization problem. However, the performance can be expected to be improved significantly. To sum up,the most appropriate solution framework should be selected by aiming at a balance between the performance and computation speed based on the design requirements. Moreover, we have tested these four solution frameworks in a simulation-based case study — the Singapore expressway network — by using two different optimization algorithms. The results show that the joint optimization, the bi-level optimization, and the iterative optimization can result in a superior performance than the separate optimization.

There are several directions in which the current research work could be extended. First, in this paper, we assume a typical daily traffic demand in the network, which, however, may differ from the real traffic demand. Therefore, it would be interesting to do the sensitivity analysis of the design solutions with respect to the uncertainty in the real traffic demand. Second, we are going to perform an extensive assessment of the four solution frameworks for various network set-ups, future demand scenarios and optimization algorithms. For future work, we will consider to use multi-objective optimization methods instead of the aggregated objective functions. Third, some other traffic models will be used for comparison, such as the cell transmission model (Daganzo, 1994), the Fastlane model (van Lint et al., 2008), the intelligent driver model (Treiber et al., 2000), and so on .

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Appendix: METANET

In this appendix, the main equations of the destination-dependent METANET model (Messmer and Papageorgiou, 1990; Kotsialos et al., 2002b,a) are provided. All symbols used below are defined in Table 4. For the sake of compactness, their definitions will not be repeated in the text.

The outflow of each segment is given by

$$q_{m,i}(k) = \rho_{m,i}(k) v_{m,i}(k) \lambda_m \ .$$

The partial traffic density in each segment for each destination is

$$\rho_{m,i,d}(k+1) = \rho_{m,i,d}(k) + \frac{T}{L_m \lambda_m} (\gamma_{m,i-1,d}(k) q_{m,i-1}(k) - \gamma_{m,i,d}(k) q_{m,i}(k)) \ .$$

The total density in each segment is

$$\rho_{m,i}(k) = \gamma_{m,i,d}(k) \rho_{m,i,d}(k) \ .$$

The speed on each segment is

$$\begin{aligned} v_{m,i}(k+1) = v_{m,i}(k) + \frac{T}{\tau}\Big(V(\rho_{m,i}(k)) - v_{m,i}(k)\Big) + \\ \frac{T}{L_m} v_{m,i}(k)\Big(v_{m,i-1}(k) - v_{m,i}(k)\Big) - \frac{\eta T}{\tau L_m} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa} \ . \end{aligned}$$

The speed-density relationship in case of variable speed limits is

$$V(\rho_{m,i}(k)) = \min\left(v_m^{\text{free}} \exp\Big[-\frac{1}{a_m} \Big(\frac{\rho_{m,i}(k)}{\rho_m^{\text{crit}}} \Big)^{a_m} \Big], (1+\alpha_m) v_{m,i}^{\text{lim}} \left(\left\lfloor \frac{k}{M} \right\rfloor \right) \right) \ .$$

The partial queue length at each origin for each destination is

$$w_{o,d}(k+1) = w_{o,d}(k) + T(\gamma_{o,d}(k) d_o(k) - \gamma_{o,d}(k) q_o(k)) \ .$$

The total queue length at each origin is

$$w_o(k) = \sum_{d \in D} w_{o,d}(k) \ .$$

The outflow of each origin is

$$q_o(k) = \min\left[d_o(k) + \frac{w_o(k)}{T}, C_o r_o \left(\left\lfloor \frac{k}{M} \right\rfloor \right), C_o \left(\frac{\rho_m^{\max} - \rho_{m,1}(k)}{\rho_m^{\max} - \rho_m^{\text{crit}}} \right) \right] \ .$$

symbol	unit	meaning
$q_{m,i}(k)$	[veh/h]	traffic outflow of segment i of link m at simulation step k
$\rho_{m,i}(k)$	[veh/km/lane]	traffic density in segment i of link m at simulation step k
$\rho_{m,i,d}(k)$	[veh/km/lane]	partial traffic density for destination d
$v_{m,i}(k)$	[km/h]	traffic mean speed for segment i of link m at simulation step k
$w_{o,d}(k)$	[veh]	partial queue length at origin o to destination d at simulation step k
$w_o(k)$	[veh]	queue length at origin o at simulation step k
$q_o(k)$	[veh/h]	outflow of origin o at simulation step k
$\beta_{n,m,d}(k)$	[-]	splitting rate of traffic flow with destination d from node n to outgoing link m at simulation step k
$\gamma_{m,i,d}(k)$	[-]	fraction of traffic flow to destination d on segment i of link m at simulation step k
$Q_{n,d}(k)$	[veh/h]	total flow to destination d that enters the node n at simulation step k
$\gamma_{o,d}(k)$	[-]	fraction of demand from origin o to destination d at simulation step k
$r_o(k_\mathrm{c})$	[-]	ramp metering rate at the origin o at control step k_c
$v_{m,i}^{\mathrm{lim}}(k_\mathrm{c})$	[km/h]	maximum allowed speed of segment i on link m at control step k_c
λ_m	[-]	number of lanes of link m
N_m	[-]	number of segments on link m
L_m	[km]	length of segments of link m
ρ_m^{max}	[veh/km/lane]	maximum density of link m
a_m	[-]	model parameter for link m
α_m	[-]	non-compliance factor of drivers for speed limits on link m
T	[s]	simulation time interval
τ	[s]	model parameter
η	[km^2/h]	model parameter
κ	[veh/km/lane]	model parameter
D	[-]	set of destinations in the traffic network
C_o	[veh/h]	on-ramp capacity under free-flow conditions
I_n	[-]	set of incoming links of node n

Table 4: Symbols used for METANET

The node equations are

$$Q_{n,d}(k) = \sum_{m' \in I_n} q_{m',N_{m'}}(k)\gamma_{m',N_{m'},d}(k)$$
$$q_{m,0}(k) = \sum_{d \in D} \beta_{n,m,d}(k) Q_{n,d}(k)$$
$$\gamma_{m,0,d}(k) = \frac{\beta_{n,m,d}(k) Q_{n,d}(k)}{q_{m,0}(k)} \ .$$

Additional details and extensions can be found in (Messmer and Papageorgiou, 1990; Hegyi et al., 2005a).

Appendix: Genetic algorithms

In a genetic algorithm, a population of candidate solutions to an optimization problem is evolved toward optimal solutions. Each candidate solution has a set of properties that can be combined and mutated. The evolution usually starts from a population of randomly generated individuals, and it is an iterative process. In each generation, the fitness of every individual in the population is evaluated, usually based on the value of the objective function in the optimization problem being solved. The best individuals are selected from the current population, and then recombined and randomly mutated to form a new generation. The new generation of candidate solutions is used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been attained, or a satisfactory fitness level has been reached for the population.

A typical genetic algorithm is shown in Algorithm 1:

Algorithm 1 Basic structure of the genetic algorithms

Input: size of total population λ, size of parent population μ, mutation probability p, maximum number of generations G

Initialization: Randomly generate a population of λ individuals at generation 0, denoted as $\mathbf{x}_\lambda^{(0)}$
for $g = 0, 1, \ldots, G-1$ **do**
  Compute fitness $f(x)$ for each $x \in \mathbf{x}_\lambda^{(g)}$
  Select μ best individuals from $\mathbf{x}_\lambda^{(g)}$ based on fitness, denoted as $\mathbf{x}_\mu^{(g+1)}$
  repeat
    Select two individuals from $\mathbf{x}_\mu^{(g+1)}$ as parents according to their fitness (the better fitness, the bigger chance to be selected)
    Cross over the parents to form new offsprings
    Mutate the new offsprings with a probability p
  until $(\lambda - \mu)$ new offsprings are generated
  Place the μ parents and the $(\lambda - \mu)$ offsprings in the next generation $\mathbf{x}_\lambda^{(g+1)}$
end for

Output: The best offspring solution at generation G: $\mathbf{x}_{\lambda,\text{best}}^{(G)}$

References

Athol, P., 1965. Interdependence of certain operational characteristics within a moving traffic stream. Highway Research Record 72, 58–87.

Bäck, T., 1996. Evolutionary algorithms in theory and practice: evolution strategies, evolutionary programming, genetic algorithms. Oxford University press, Oxford, UK.

Bellemans, T., De Schutter, B., De Moor, B., 2006. Model predictive control for ramp metering of motorway traffic: A case study. Control Engineering Practice 14, 757–767.

Belotti, P., Kirches, C., Leyffer, S., Linderoth, J., Luedtke, J., Mahajan, A., 2013. Mixed-integer nonlinear optimization. Acta Numerica 22, 1–131.

Bezdek, J.C., Hathaway, R.J., 2002. Some notes on alternating optimization, in: Nikhil, R.P., Michio, S. (Eds.), Advances in Soft Computing—AFSS 2002. Springer Berlin Heidelberg, Calcutta, India, pp. 288–300.

Bogers, E., Viti, F., Hoogendoorn, S., 2005. Joint modeling of advanced travel information service, habit, and learning impacts on route choice by laboratory simulator experiments. Transportation Research Record , 189–197.

Boggs, P.T., Tolle, J.W., 1995. Sequential quadratic programming. Acta Numerica 4, 1–51.

Boyd, S., Parikh, N., Chu, E., Peleato, B., Eckstein, J., 2011. Distributed optimization and statistical learning via the alternating direction method of multipliers. Foundations and Trends in Machine Learning 3, 1–122.

Braess, D., Nagurney, A., Wakolbinger, T., 2005. On a paradox of traffic planning. Transportation Science 39, 446–450.

Candler, W., Townsley, R., 1982. A linear two-level programming problem. Computers and Operations Research 9, 59–76.

Cantarella, G.E., Pavone, G., Vitetta, A., 2006. Heuristics for urban road network design: Lane layout and signal settings. European Journal of Operational Research 175, 1682–1695.

Chen, M., Alfa, A.S., 1991. A network design algorithm using a stochastic incremental traffic assignment approach. Transportation Science 25, 215–224.

Chiou, S., 2005. Bilevel programming for the continuous transport network design problem. Transportation Research Part B: Methodological 39, 361–383.

Daganzo, C., 1994. The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. Transportation Research Part B 28, 269–287.

DeSerpa, A.C., 1971. A theory of the economics of time. The Economic Journal 81, 828–846.

Duran, M.A., Grossmann, I.E., 1986. An outer-approximation algorithm for a class of mixed-integer nonlinear programs. Mathematical programming 36, 307–339.

Eiben, A., Hinterding, R., Michalewicz, Z., 1999. Parameter control in evolutionary algorithms. IEEE Transactions on Evolutionary Computation 3, 124–141.

Friesz, T.L., Tobin, R.L., Cho, H.J., Mehta, N.J., 1990. Sensitivity analysis based heuristic algorithms for mathematical programs with variational inequality constraints. Mathematical Programming 48, 265–284.

Gao, Z., Wu, J., Sun, H., 2005. Solution algorithm for the bi-level discrete network design problem. Transportation Research Part B: Methodological 39, 479–495.

Geoffrion, A.M., 1972. Generalized benders decomposition. Journal of Optimization Theory and Applications 10, 237–260.

Hadjiconstantinou, E., Christofides, N., 1999. An efficient implementation of an algorithm for finding K shortest simple paths. Networks 34, 88–101.

Hegyi, A., De Schutter, B., Hellendoorn, H., 2005a. Model predictive control for optimal coordination of ramp metering and variable speed limits. Transportation Research Part C 13, 185–209.

Hegyi, A., De Schutter, B., Hellendoorn, J., 2005b. Optimal coordination of variable speed limits to suppress shock waves. IEEE Transactions on Intelligent Transportation Systems 6, 102–112.

Hooke, R., Jeeves, T.A., 1961. "direct search" solution of numerical and statistical problems. Journal of the ACM (JACM) 8, 212–229.

Hoppe, H., DeRose, T., Duchamp, T., McDonald, J., Stuetzle, W., 1993. Mesh optimization, in: Proceedings of the 20th Annual Conference on Computer Graphics and Interactive Techniques, Anaheim, USA. pp. 19–26.

Hu, T.Y., Mahmassani, H.S., 1997. Day-to-day evolution of network flows under real-time information and reactive signal control. Transportation Research Part C: Emerging Technologies 5, 51–69.

Katoh, N., Ibaraki, T., Mine, H., 1982. An efficient algorithm for K shortest simple paths. Networks 12, 411–427.

Kirkpatrick, S., Gelatt, C.D., Vecchi, M.P., 1983. Optimization by simmulated annealing. Science 220, 671–680.

Kotsialos, A., Papageorgiou, M., 2004. Efficiency and equity properties of freeway network-wide ramp metering with amoc. Transportation Research Part C: Emerging Technologies 12, 401–420.

Kotsialos, A., Papageorgiou, M., Diakaki, C., Pavlis, Y., Middelham, F., 2002a. Traffic flow modeling of large-scale motorway networks using the macroscopic modeling tool METANET. IEEE Transactions on Intelligent Transportation Systems 3, 282–292.

Kotsialos, A., Papageorgiou, M., Mangeas, M., Haj-Salem, H., 2002b. Coordinated and integrated control of motorway networks via non-linear optimal control. Transportation Research Part C: Emerging Technologies 10, 65–84.

Lawler, E.L., Wood, D.E., 1966. Branch-and-bound methods: A survey. Operations Research 14, 699–719.

van Lint, J., Hoogendoorn, S., Schreuder, M., 2008. Fastlane: New multiclass first-order traffic flow model. Transportation Research Record , 177–187.

Lobo, F., Lima, C., Michalewicz, Z., 2007. Parameter Setting in Evolutionary Algorithms. Springer.

Maciejowski, J.M., 2002. Predictive Control with Constraints. Prentice Hall, Harlow, England.

Magnanti, T.L., Wong, R.T., 1984. Network design and transportation planning: Models and algorithms. Transportation Science 18, 1–55.

Mahmassani, H., Huynh, N., Srinivasan, K., Kraan, M., 2003. Tripmaker choice behavior for shopping trips under real-time information: Model formulation and results of stated-preference internet-based interactive experiments. Journal of Retailing and Consumer Services 10, 311–321.

Messmer, A., Papageorgiou, M., 1990. METANET: A macroscopic simulation program for motorway networks. Traffic Engineering and Control 31, 466–470.

Middelham, F., 2003. State of practice in dynamic traffic management in the netherlands, in: Proceedings of the 10th IFAC Symposium on Control in Transportation Systems (CTS 2003), Tokyo, Japan.

Papageorgiou, M., Blosseville, J.M., Hadj-Salem, H., 1998. La Fluidification des Rocades de IIle de France: Un Projet dImportance. Internal report No. 1998-17. Dynamic Systems and Simulation Laboratory, Technical University of Crete. Chania, Greece.

Papageorgiou, M., Diakaki, C., Dinopoulou, V., Kotsialos, A., Wang, Y., 2003. Review of road traffic control strategies. Proceedings of the IEEE 91, 2043–2067.

Papageorgiou, M., Hadj-Salem, H., Blosseville, J., 1991. ALINEA: A local feedback control law for on-ramp metering. Transportation Research Record , 58–64.

Papageorgiou, M., Kosmatopoulos, E., Papamichail, I., 2008. Effects of variable speed limits on motorway traffic flow. Transportation Research Record: Journal of the Transportation Research Board 2047, 37–48.

Papageorgiou, M., Kotsialos, A., 2002. Freeway ramp metering: An overview. IEEE Transactions on intelligent transportation systems 3, 271–280.

Poorzahedy, H., Turnquist, M.A., 1982. Approximate algorithms for the discrete network design problem. Transportation Research Part B: Methodological 16, 45–55.

Sheffi, Y., 1985. Urban transportation networks: Equilibrium analysis with mathematical programming methods. Prentice-Hall, Englewood Cliffs, New Jersey.

Shin, Y.C., Joo, Y.S., 1992. Optimization of machining conditions with practical constraints. The International Journal of Production Research 30, 2907–2919.

Smulders, S., 1990. Control of freeway traffic flow by variable speed signs. Transportation Research Part B: Methodological 24, 111–132.

Tawarmalani, M., Sahinidis, N.V., 2005. A polyhedral branch-and-cut approach to global optimization. Mathematical Programming 103, 225–249.

Tobin, R.L., Friesz, T.L., 1988. Sensitivity analysis for equilibrium network flow. Transportation Science 22, 242–250.

Treiber, M., Hennecke, A., Helbing, D., 2000. Congested traffic states in empirical observations and microscopic simulations. Physical Review E 62, 1805–1824.

U.S. General Accounting Office, 2003. Comparison of States' Highway Construction Costs. Technical report.

Wardman, M., 1998. The value of travel time: a review of british evidence. Journal of Transport Economics and Policy 32, 285–316.

Williams, B., 1996. Highway control. IEE Review 42, 191–194.

Yang, H., 1997. Sensitivity analysis for the elastic-demand network equilibrium problem with applications. Transportation Research Part B: Methodological 31, 55–70.

Yang, H., Bell, M.G.H., 1998. Models and algorithms for road network design: a review and some new developments. Transport Reviews 18, 257–278.

Yen, J.Y., 1971. Finding the K shortest loopless paths in a network. Management Science 17, 712–716.

Zegeye, S.K., De Schutter, B., Hellendoorn, J., Breunesse, E.A., Hegyi, A., 2012. A predictive traffic controller for sustainable mobility using parameterized control policies. IEEE Transactions on Intelligent Transportation Systems 13, 1420–1429.